Economic growth with endogenous labour supply

Clas Eriksson

E-inst., University College of Gävle / Sandviken, 801 76 Gävle, Sweden

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Abstract

In this paper versions of a standard growth model are extended to capture disutility from labour. One effect is that the steady-state growth rate is affected by preference parameters, when technology is endogenously produced and also when exogenously given. In these models, changed attitudes to work can affect the growth of GNP, capital and consumption in quite predictable ways. The effect on the endogenous productivity growth is however ambiguous, and depends on the complementarity of labour and the technology factor in the sector where the latter is produced.

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1. Introduction

The choice between work and leisure has been remarkably neglected in the theory of economic growth. For example, in Burmeister and Dobell (1970), Solow (1970), Wan (1971), Dixit (1976) and Grossman and Helpman (1991) there is no
attempt to model this relation. The standard argument for this omission is that the theory of economic growth is intended for studies of long-run development which do not focus on fluctuations in employment. Yet there is an important long-run aspect of labour supply. Technological progress which makes labour more productive and increases wages obliges agents to decide how to allocate the fruits between increased leisure and increased consumption.

This omission has been pointed out by Hahn (1991), who also notes that working hours have fallen historically. He therefore supplements the standard exogenous growth model with a utility function that has both consumption and leisure as arguments. However, since he uses a Cobb–Douglas function, labour supply is constant over time.

In this paper I allow for disutility from work in three versions of a mainstream growth model, which does not require that labour supply be constant in steady state. If there is a relatively tendency to satiation in consumption, that is if the elasticity of intertemporal substitution in consumption is smaller than one (for which Hall (1988) supplies some empirical support) labour supply falls over time, but not fast enough for production and consumption to fall. An interesting implication is that what seems to be a growth problem does not have to be a problem; sudden decreases in the growth rates of production, consumption and capital may, according to this model, be caused by preference shifts. Of course, I do not argue that we cannot see any growth problems around us, but I believe that we should contemplate whether the decline in growth rates in many countries to some extent reflects our preferences.

Changed preference parameters can thus trigger a growth slowdown in the models presented here. Can they also trigger a productivity slowdown? I analyse this question in Section 3, where technology is assumed to be endogenously produced. The answer depends on whether or not labour and the technology factor

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1 However, the classical articles by Ramsey (1928) and Solow (1956) contain more on this topic than is perhaps commonly believed. Ramsey's paper includes a condition that will be central in the models of this paper: the marginal disutility of labour must be equal to the marginal product of labour times the marginal utility of consumption at every instant. Unfortunately, he does not follow this up by a thorough analysis of the labour supply. Solow suggests that his model can be supplemented by a function that makes labour supply proportional to the real wage. This relationship is not explicitly derived from first principles, and he only seems to have a positive correlation between these two variables in mind. In this paper we will mostly discuss the opposite. Finally, it should be noted that Barro and Sala-i-Martin (1995) contains a section on endogenous labour supply.

2 His analysis also indicates that deviations from this constant level of labour supply will have highly implausible consequences: It will be optimal to work more than 24 hours or less than 0 hours a day, in finite time.

3 This would be consistent with the sociological theories about the rise of 'post-materialist values' as people start to feel that they are no longer living under 'scarcity'. See Inglehart (1990, ch. 2).
are complements in the technology-producing sector. A higher sensitivity of labour to its compensation affects productivity growth positively when labour and the technology factor are not complements.

Throughout this paper I will take it to be a stylised fact that working time has declined over time. Pencavel (1986) states that for “a century or so, at least in North America and West Europe, a declining fraction of a man’s lifetime has been spent at market work” (p. 7). Men’s and women’s changes in labour force participation rates have largely offset each other, at least in some countries. However, the number of weekly hours worked has declined for both sexes (cf. Killingsworth and Heckman, 1986). Hence it would appear that the time worked by a representative person has fallen. There are also some indications that total working time (‘market’ and ‘non-market’) has been declining for some decades (cf. Juster and Stafford, 1991) The discussion in this paper will therefore be based on the assumption that there has been a trend decline in labour supply. 4 In the model this is entirely due to the monotonously rising productivity of labour. 5

Although pure growth theorists have ignored disutility of labour issues, these issues have been analysed in related contexts. The questions raised differ from those which concern us here, but it is appropriate to mention them. First there is the literature on real business cycles. For these models, the authors typically choose utility functions such that allocations of labour and leisure are constant in steady state. For examples, see Rebelo (1991). The time perspective in those papers is shorter than in my model. Second, there is a vast literature on dynamic labour supply (and education) with a partial approach. Such models differ from growth models by having a finite time horizon, an exogenously given interest rate and a (growing) wage rate that often is independent of labour supply. 6 Third, some of the models on optimal taxation in dynamic economies come close to those set up here, but there still remain considerable differences. For example, in Chamley (1981) the utility function is chosen so as to preclude any trend in labour supply, or there is no technological progress at all, as in Chamley (1986). The analysis of this paper differs from this literature in the focus on general equilibrium and the long-term development.

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4 I interpret the recent upturns, reported for example by Juster and Stafford (1991), to be cyclical rather than long-run phenomena.
5 Concerning Swedish post-war conditions, I have computed a time series by dividing the total number of hours worked by the number of persons aged 16 to 74. (The sources are the National Accounts and Vital Statistics, published annually by Statistics Sweden.) The series reveals a weak trend decline in hours worked.
6 Notable examples are Blinder and Weiss (1976) and Ryder et al. (1976). For a survey, see Weiss (1986).
2. Exogenous technological progress

In this section, I analyse the first version of a growth model in which the labour supply, of a constant population, is endogenously determined. In order to attain analytical tractability, the instantaneous utility function is additively separable in consumption \( c(t) \) and (disutility of) labour \( l(t) \), and both terms are constant-elastic. Thus the function is

\[
\nu(c(t), l(t)) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}, \quad \sigma > 0, \quad \sigma \neq 1, \quad \gamma > 0. \tag{1}
\]

The properties of the first term are well known. Concerning the second term, \(-1/\gamma\) is the intertemporal elasticity of substitution of labour. If \( \gamma \) is small, the function is relatively flat, which implies that there is a substantial propensity to alter the amount of labour supplied as the returns to it varies. Conversely, as \( \gamma \) increases, \(-l^{1+\gamma}/(1+\gamma)\) becomes more concave, and the flexibility of labour supply decreases; one becomes more prone to smooth disutility of labour over time.

There are no externalities present, so the market solution is efficient. For ease of exposition I let a benevolent planner replicate market outcomes and take all decisions, so as to maximise the discounted stream of utilities enjoyed by every identical agent in the economy. The objective is thus to maximise

\[
\int_0^\infty e^{-\rho t} \left[ \frac{c(t)^{1-\sigma}}{1-\sigma} - \frac{l(t)^{1+\gamma}}{1+\gamma} \right] dt \tag{2}
\]

where \( c \) and \( l \) are control variables and \( \rho \) is the discount factor. There is also a resource constraint, implying that consumption plus investment exactly exhaust output (net of depreciation) at every instant. Assuming a Cobb–Douglas production technology, it takes the form

\[
\dot{K}(t) = K(t)^{1-\beta} \left[ a(t) l(t) \right]^{\beta-1} - c(t). \tag{3}
\]

Here \( K(t) \) represents the amount of capital, and a dot over a variable denotes the time derivative. There is also labour-augmenting, exogenous technological progress, captured by \( a(t) \), which is assumed to grow exponentially at the rate \( \alpha \). As \( K \) and \( a \) grow, this implies that the marginal product of labour is increasing over time. The price of the consumption good/capital is normalised to unity. To see how the economy reacts to this change in ‘relative price’, by consumption and labour supply decisions, is the central purpose of this paper. From here on I suppress the time dependence of the variables.

The economic problem is thus to maximise (2) subject to (3), given the initial capital stock \( K_0 \). This is a standard optimal control problem (see the working paper version of this paper (Eriksson 1993) available on request)). It is worth noting that, because \( l \) is a control variable, we have the additional first-order
condition \( l^\gamma = c^{-\sigma} (1 - \beta) K^{\beta a^{1-\beta}} l^{-\beta}, \) which implies that the marginal disutility of labour should equal the value (= marginal utility of consumption) of output produced by a marginal work effort.\(^7\)

This model is saddle point stable, and satisfies the transversality condition when the economy converges to the steady state. It turns out that the steady-state rate of change of labour is

\[
\hat{l} = \frac{1 - \sigma}{\sigma + \gamma} \alpha
\]

(4)

where the circumflex denotes the proportional rate of change \((dl/dt)/l\). The upshot is therefore that the value of \(\sigma\) is crucial for the qualitative behaviour of labour supply over time. Given the stylised fact of a falling labour supply over longer periods of time, referred to in the introduction, we are mainly interested to see what it takes for this model to display a negative trend for this variable. The answer is obviously that \(\sigma\) must be larger than one. To assume this does not seem to be in conflict with existing empirical evidence.\(^8\) If this is the case, \(l\) will fall asymptotically towards zero.

The interpretation of this is straightforward: a large \(\sigma\) means a high 'relative concavity' of the part of the utility function that captures utility from consumption. In fact, this concavity is so strong that the utility of consumption asymptotically goes towards a satiation level; \(c^{1-\sigma}/(1 - \sigma)\) approaches zero from below as \(c\) increases. Hence, in this case, agents' value of incremental consumption is so low that only a modest share of the fruits of technological progress is devoted to it. Instead, working time is reduced.

The solution to this model also implies that \(\hat{K} = \hat{\sigma} = \alpha + \hat{l}\). We can therefore obtain

\[
\hat{K} = \hat{\sigma} = \alpha + \hat{l} = \frac{1 + \gamma}{\gamma + \sigma} \alpha > 0.
\]

(5)

Hence this model describes an economy in which capital, consumption and efficient labour supply always grow in steady state. As long as labour supply is declining, this growth rate is however smaller than the rate of technological change, because a part of the latter must be used to 'pay' for the release from work. It is interesting to note that, unlike the standard model, preferences here affect the steady-state growth rate. This is a property that the present model has in common with models of endogenous growth. A difference, though, is that the time preference \(\rho\) does not play any role in the determination of the growth rate here.

From Eq. (4) and Eq. (5) we see that if the exogenous rate of technological progress \((\alpha)\) increases, the economy can afford to accumulate capital, increase

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7 For a similar expression, confer Eq. (2) in Ramsey (1928).
consumption and decrease labour effort at faster rates. If there is a hike in \( \sigma \), then \( \dot{K}, \dot{c} \) and \( \dot{l} \) become smaller. A larger \( \sigma \) means that the utility of consumption will approach the asymptotic satiation level faster than before. The propensity to use the fruits of technological progress foremost for relief from work then becomes more conspicuous. Given this faster reduction of one of the production factors, the returns to capital decrease and therefore capital accumulation is retarded.

Finally, as \( \gamma \) increases, so do the growth rates of \( K, c \) and \( l \). As noted before, when \( \gamma \) becomes larger, the responsiveness of labour supply to changes in returns to labour decreases. Hence \( l \) falls at a slower rate as its marginal product grows, and therefore there are resources available to let the capital stock and consumption grow faster.

This model thus gives some ideas about how an economy, and in particular the labour supply, reacts in response to technological progress. Things are different when technology improves only if scarce resources are allocated for this purpose. Then there are some interesting interactions between labour supply and technological progress. We now turn to a model in which this is the case.

### 3. Endogenous technological progress

I will now make a slight modification of the model of the preceding section, so that it becomes quite similar to the models in Uzawa (1965) and Lucas (1988), with the important exception that I retain the term capturing disutility of labour. On the production side of the economy, two new variables are introduced. First, instead of \( a \), we have \( h \) for (endogenously created) human capital. Second, there is one unit of effort, attached to the human capital, to be allocated between the production of the consumption good/capital and human capital respectively. If we denote the share allotted to the former by \( s \), real capital now accumulates according to

\[
\dot{K}(t) = K(t)\beta \left[ s(t)h(t)l(t) \right]^{1-\beta} - c(t), \quad 0 < \beta < 1. \tag{6}
\]

In order to endogenize the rate of growth of human capital, we will use the same relationship as Uzawa (1965) and Lucas (1988). The change of human capital is a function of the stock and the effort used to acquire more:

\[
\dot{h}(t) = \delta h(t)(1 - s(t)). \tag{7}
\]

The constant \( \delta \) captures the productivity of this sector. Since this function is linear in \( h \), there are no diminishing returns to human capital. Therefore the model exhibits unceasing growth, on the condition that \( \delta \) is large enough.

Since \( l \) and \( h \) serve as two distinct variables in this model, we have to think of them as two distinct kinds of work, performed by different persons or by the same persons at different times of the day. The work represented by \( l \) entails some disutility, whereas there is no disutility associated with the work that requires
human capital. One can motivate this by saying that performing tasks for which special skills are needed, is fascinating and exciting, which thereby compensates for possible disutility. 9

Our problem is thus to maximise (2) with respect to \(c, l\) and \(s\), subject to (6) and (7) and initial values of \(K\) and \(h\). The steady-state reaction of labour supply to changes in human capital is now described by an expression quite similar to Eq. (4) in the previous section:

\[
\hat{l} = \frac{1 - \sigma}{\sigma + \gamma} \hat{h}
\]

(8)

When \(h\) is growing, the condition for a falling labour supply is again that \(\sigma > 1\), with the same explanation as previously. I will retain the assumption that this inequality holds. In terms of the model’s parameters, the variables change as follows:

\[
\hat{h} = \frac{\sigma + \gamma}{2\sigma + \sigma\gamma - 1}[\delta - \rho] > 0,
\]

(9)

\[
\hat{l} = \frac{1 - \sigma}{2\sigma + \gamma\sigma - 1}[\delta - \rho] < 0,
\]

(10)

\[
\hat{y} = \hat{\dot{K}} = \hat{\dot{c}} = \hat{h} + \hat{l} = \frac{1 + \gamma}{2\sigma + \sigma\gamma - 1}[\delta - \rho] > 0.
\]

(11)

I have here determined the signs under the assumption that \(\delta - \rho > 0\), i.e. that the human capital sector is productive enough for optimal growth to be positive.

The responses of the steady-state growth rates 10 to changed parameter values are in many respects qualitatively the same as in most endogenous growth models (cf. Barro and Sala-i-Martin (1995)). The novelties are due to the presence of \(\hat{l}\) and \(\gamma\), so it suffices here to consider the consequences of changes in \(\gamma\). The growth rates are affected as follows: \(\delta\hat{l}/\delta\gamma = \delta\hat{c}/\delta\gamma > 0\) and \(\delta\hat{h}/\delta\gamma < 0\). As before, when \(\gamma\) increases, labour supply is less sensitive to increased returns to labour. This means that the supply of labour decreases at a slower rate. As a consequence, consumption and production can be increased at higher rates. More surprising, perhaps, is that these changes coincide with a slower growth rate of human capital. The reason is that labour and effort of human capital are complementary in the production of goods but not in the production of human capital. More labor means higher returns to human capital in the commodity sector but not in the human capital sector. Therefore some effort is reallocated away from the latter, the growth rate of which is then lower in steady state.

9 For empirical evidence supporting this assumption see Juster and Stafford (1985).

10 Because we have two state variables in the present model, a study of the transitional dynamics is difficult. However, based on the results in Mulligan and Sala-i-Martin (1993) (where related models are studied), it is likely that the present model converges (or jumps) to the steady state.
The effect just described is a plausible albeit not the only plausible one. It is obvious that the formulation of Eq. (7) is crucial for this result; had the production function of human capital contained \( l \) as an argument, so that \( l \) and \( h \) would have been complementary in this sector too, the derivative of \( h \) with respect to \( \gamma \) would have been ambiguous. Such a modification would however give rise to considerable complications, and I will instead use the next section to supplement this model with a much simpler version, in which at least \( l \) and \( h \) are complementary in the sector where the latter is produced.

4. Learning-by-doing

In many recent growth models, the growth rates depend positively on the (constant) amounts of labour supplied. In the Arrow–Romer learning-by-doing model described in Romer (1989), the reason is that a larger amount of labour makes the returns to capital higher, by complementarity in production. The R&D growth model with imperfect competition presented in Barro and Sala-i-Martin (1992) exhibits the same property because when \( l \) increases, market size increases. Then the once-and-for-all cost of research for a specific R&D output can be spread across a larger market, and the incentives to undertake research improve. This results in a higher growth rate.

This positive correlation between size and growth rate is embarrassing for these theories, because it is not consistent with the data. \(^{11}\) I take it, however, that the mechanisms described in the previous paragraph do exist, although they have too strong impacts in the outcomes of the models. Therefore I find it motivated to accept this phenomenon in the model to which we now turn. It is intended to describe the interaction between preferences and productivity in the simplest possible way. To this end, I draw on the quasi-growth models in Agell and Lommerud (1991), Krugman (1987) and Lucas (1988, sctn. 5), in which there are no investment problems, and technological progress occurs only through economy-wide externalities whose magnitudes no single agent can influence.

The optimising agent is no longer a benevolent social planner, but a yeoman farmer. As before, utility accrues from consumption and disutility from labour, but the function describing this is now slightly more general,

\[
U = \int_0^\infty [u(c) - v(l)]e^{-\rho t}dt
\]

(12)

where \( u(c) \) is concave (with \( \lim_{c \to 0} u'(c) = \infty \)) and \( v(l) \) is convex. By means of

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\(^{11}\) See Jones (1995). He also provides an interesting model that does no exhibit this unpleasant property.
labour and human capital, every agent produces the consumption good according to the production function

\[ y = hl. \]  

(13)

This function is perceived to be just linear in the individual's single decision variable \( l \), since we will assume that a single agent's behaviour cannot have an effect on the size of \( h \).

Over time, this society increases its wealth because there is a learning-by-doing effect associated with the production process, which enhances the stock of human capital. For convenience, I follow the authors mentioned above, by assuming that this effect is entirely external to each agent. The functional form that will be used to capture this process is also quite common,

\[ h = hf(l_a), \quad f'(l_a) > 0, \]  

(14)

where \( l_a \) is the average labour supply, which no agent can affect. Therefore this expression will be redundant when we derive the first-order condition for optimality.

Because there is no saving, one can substitute Eq. (13) for \( c \) into Eq. (12) and maximise with respect to \( l \). We obtain the first-order condition \( hu'(hl) - v'(l) = 0 \forall t \). Differentiating this expression with respect to time, and rearranging, we arrive at another expression similar to Eq. (4):

\[ \gamma l = \frac{1 - \sigma(c)}{\gamma(l) + \sigma(c)} \hat{h} \]  

(15)

where \( \sigma(c) = -u'(c)c/u'(c) > 0 \) and \( \gamma(l) = u''(l)l/v' > 0 \). We thus obtain the same condition for a falling labour supply as in the two previous sections: \( \sigma(c) \) must be larger than one. This again means that the relative concavity of \( u(c) \) is significant and there is a relatively strong tendency to satiation.\(^{12}\)

In this model it is natural to define the rate of productivity growth as \( \dot{h} = \hat{h} = f(l) \), which is positively correlated with \( l \). Thus if \( \sigma(c) \) is larger than one, the model predicts not only successively less work, but also a slowdown of productivity growth.\(^{13}\)

The reason for concern about productivity in recent years is however not that its growth rate has declined monotonously since the beginning of industrialization, but rather that there has been a sharp decline since the beginning of the 1970's. Without proposing this to be the sole actual reason, we can let preferences initiate a productivity slowdown in this model. What we need to assume is that the

\(^{12}\) By the assumptions about \( u(c) \), \( c = hl \) will always be strictly positive, and by Eq. (14) \( h \) will always grow.

\(^{13}\) Having taken the first-order condition for utility maximisation, we no longer distinguish between \( l \) and \( l_a \), since agents are assumed to be identical.
elasticity of intertemporal substitution varies with consumption in such a way that \(\sigma(c) = 1\) for consumption levels lower than some \(c^*\) and \(\sigma(c) > 1\) for \(c \geq c^*\). That is, there is a greater tendency to satiation at higher levels of consumption. By (15) we will thus have an era of constant labour supply and productivity growth, succeeded by an era of declining labour supply and productivity growth rate. In some sense, then, this productivity slowdown can be regarded as chosen, although we must remember that there is an externality that is not internalised in this model. If it were, we could probably still have decreases of \(l\) and \(h\), but at slower rates. As long as there are positive externalities, there is of course scope in principle for policy measures that internalise them and increase welfare, \(^{14}\) if the attempt to correct market failure does not meet with ‘political failure’.

5. Conclusions

In this paper I have examined the properties of three versions of a standard growth model that make allowance for disutility of labour. The first was shown to mimic the Solow–Cass–Koopmans model in many respects. A notable difference was that the natural growth rate (obtained in the steady state) was not merely equal to the exogenous rate of technological change, but contained also an endogenous component, consisting of preference parameters. As one could expect, whether labour supply will decrease or increase over time depends critically on the properties of the utility function. The model is saddle point stable.

The introduction of endogenous technological progress did not alter the condition for a falling trend in labour supply. The interaction between these two variables differed however, depending on whether they were complementary in the technology sector or not. Further research is required to give more clarity here, by introducing more margins in the models. Unfortunately, one will then probably have to be content with numerical studies. Nevertheless, such approaches should be able to give insights into answers to important economic questions. For example, it is not unlikely that labour supply gives rise to positive externalities, thereby increasing the incentives to undertake costly technological research. If this is so, the line of research pursued in this paper should lead to indications on how to internalise the externalities.

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\(^{14}\) Combining \(\hat{y} = \hat{c} = \hat{h} + \hat{\ell}\) with Eq. (15) one will see that \(\hat{c}\) and \(\hat{y}\) are always positive. That is, labour supply will never decrease so rapidly so rapidly that consumption and production fall.
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