

This exam consists of five problems, each of which is worth 4 points. Marks:

Points	ECTS mark	Swedish mark
19-20	A	VG
18	B	VG
15-17	C	G
12-14	D	G
9-11	E	G

1. Evaluate the limit or explain why it does not exist.

$$\lim_{x \rightarrow 0} \frac{\sin(x^2)}{1 - \cos(x)}.$$

2. Suppose that

$$y + y^2 = e^x.$$

Find an expression for $\frac{dy}{dx}$. Write the answer in a form that does not contain any exponential functions.

3. Evaluate the definite integral.

$$\int_0^2 \frac{x}{\sqrt{x+1}} dx$$

4. Find the solution of the (separable) differential equation that satisfies the boundary condition.

$$y' = y^2 \cdot \sin(x), \quad y(0) = \frac{1}{2}.$$

5. Consider the following power series:

$$\sum_{n=0}^{\infty} (-1)^n e^{n-\sqrt{n}} x^n.$$

Determine its radius of convergence.

Good luck!