

This exam consists of five problems, each of which is worth 4 points. Marks:

Points	ECTS mark	Swedish mark
19–20	A	VG
18	B	VG
15–17	C	G
12–14	D	G
9–11	E	G

1. Evaluate the limit or explain why it does not exist.

$$\lim_{x \rightarrow 0} \frac{x^2 - x}{e^x - 1}.$$

2. Let $f(x) = e^x(x^2 + \sin(x))$.

(a) Find $f'(x)$ (1 p.)

(b) Find $f''(x)$ (3 p.)

3. Evaluate the definite integral.

$$\int_1^e \frac{\ln(x)}{x} dx$$

4. Find the solution of the differential equation that satisfies the boundary condition.

$$y' + \sin(x)y = e^{\cos(x)}, \quad y(0) = 1.$$

5. Consider the following power series:

$$\sum_{n=0}^{\infty} \sqrt{n} e^{-n} x^n.$$

Determine its radius of convergence.

Good luck!