1

As a preliminary step we compute

$$\lim_{x\to\infty} \frac{x+\sin((x)}{x} = \lim_{x\to\infty} 1 + \frac{\sin(x)}{x} = 1 + 0 = 1.$$

Since arctan is a continuous function we get

$$\lim_{x\to\infty} \arctan\left(\frac{x+\sin(x)}{x}\right) = \arctan(1) = \frac{\pi}{4}$$

Answer: $\frac{\pi}{4}$

2

We compute the derivative wrt x of both sides and find that

$$y' \cdot (1 + \tan^2(y)) = 1 + y'.$$

This is equivalent to

$$y' \cdot \tan^2(y) = 1. \tag{1}$$

Since we know that $\tan(y) = x + y$ we insert this into our equation (1). We find that

$$y' \cdot (x+y)^2 = 1,$$

or

$$y' = \frac{1}{(x+y)^2}$$

Answer: $y' = \frac{1}{(x+y)^2}$.

3

We use the substitution $x = u^2$. We find that $\frac{dx}{du} = 2u$ and that the new limits of integration are 0 and 4. Thus,

$$\int_0^4 \frac{\sqrt{x}}{x+1}\,dx = \int_0^2 \frac{2u^2}{u^2+1}\,du = 2\int_0^2 \frac{u^2+1}{u^2+1}\,du - 2\int_0^2 \frac{1}{u^2+1}\,du =$$
$$= 4 - 2[\arctan(u)]_0^2 = 4 - 2\arctan(2).$$

Answer: $4 - 2\arctan(2)$.

4

An integrating factor is $e^{\sin(x)}$. We multiply the equation by this factor and rewrite.

$$e^{\sin(x)}y' + e^{\sin(x)}y = 0 \Leftrightarrow (e^{\sin(x)}y)' = 0 \Leftrightarrow e^{\sin(x)}y = C \Leftrightarrow y = Ce^{-\sin(x)}.$$

Answer: $y = Ce^{-\sin(x)}$, where C is an arbitrary constant.

5

We apply the ratio test. Set $a_n = n^2x^n$. Then

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{(n+1)^2}{{}^2n} \cdot |x| = \lim_{n\to\infty} (1 + \frac{1}{n})^2 \cdot |x| = 1^2 \cdot |x| = |x|.$$

By the ratio test the series converges if $|x| < 1$ and diverges if $|x| > 1$.

Answer: The radius of convergence is 1.