List of topic suggestions for projects in MAA704 Applied matrix analysis

The project consists of a written report of approximately $5 - 15$ pages and an oral presentation. Here is a list of suggested topics which you can choose, it is also possible to propose another topic for your project. The projects are to be done in groups of $2 - 3$ people. If you have not chosen a group and a topic before the 21st of November you will be assigned a group and topic by the teachers.

The presentations should be 15 minutes long and given during the time for project presentations on the 19th of December.

It is recommended to send a draft of your project report to the teachers before the 12th of December so you have the possibility to get some feedback on your project. The final report are to be submitted at the latest 11th of January.

The report and presentation can give a maximum of 20 points. Both the report and presentation are required to pass the project, furthermore the report and presentation must be awarded at least 10 points.

While the topic suggestions are grouped under different sections, you can choose any topic you want. Additionally we note that the suggestions under "General" contains many topics that could fit under multiple fields depending on how you choose to do it.

You are free to use (and sometimes more or less required) to use computer software such as Matlab or Maple in your projects as long as you properly explain what you do and why.

Some guidelines to writing your report

Here are a couple of guidelines which can be useful to keep in mind while writing your report.

- Make sure to always cite your sources and include your references in the bibliography at the end. While we do not have any required bibliography style, make sure that your references at least include author(s), title and year, as well as time of access for any Internet sources.
- Write your own text and do not copy from somewhere else. Be especially wary of including images or graphs directly from a source, this is usually not ok unless you have the artists/authors permission or it is published under a license permitting anyone to use it. A good idea can be to first write the titles of all your sections and a couple of words of what you want to include in each before you start writing. If you see your sources as a reminder if you forgot some detail or need to check if something is correct rather than something to be followed to the letter you are on the right track.
- Keep your notation clear and consistent throughout the whole report. Regardless of whether you choose to write in LaTeX or Word or something else, equations and matrices should be written by yourself and not included as images from a print screen or similar. LaTeX is very good for writing mathematical text, but it can be done reasonably fast in Word nowadays as well.
- If you project topic is about a method or algorithm it is a very good idea to try and implement and test it yourself. If you cannot implement it you should make sure to in addition to the explanation of the method itself also give an indepth example of the method on some made up data, such an example should be a different one than any example used in your sources in order to show that you have understood the method.
- Ask the teachers if you get stuck! There is usually some time both during the lectures as well as the possibility to book a meeting with one of the teachers.

Contents

1 General

Here we present projects that either doesn't fit in under any of the other sections, or more general projects that fit under multiple of them.

1.1 Principal Components Analysis

In many research fields efficient methods of collecing data have been developed. But in order to make any real progress this data need to be analyzed. A common method for data analysis, originally developed by Karl Pearson in 1901, is *principal component analysis* (PCA). PCA is useful in situations where there is a large set of random variables and it is not clear which of these variables are most important or how they relate to each other. The goal of the technique is to find a smaller set of linear combinations of variables that give a good (but usually not complete) description of the systems behaviour. This both reduces the amount of that that needs to be analyzed and can help greatly in interpreting the results of measurements upon a system.

Mathematically PCA is very closely related to the singular value factorization of covariance matrices and is sometimes referred to as *proper orthogonal decomposition* (POD) in mathematical text. A possible project would be taking a closer look at this relation and discuss some of the possible mathematical interpretations of the method.

As PCA is a method for data analysis it is relvant in many different fields of research, it is applicable to analysing stock prices, the behaviour of groups of people, how the energy consumption of a building is dependent on outside factors, how pollution from different sources affects the concentration of toxins in a lake and how the different kinds of fuel quality affect the efficiency of a burner. A shorter and less thorough description of PCA and application to an interesting data set would also be a good project.

Two good sources on PCA in general is A Tutorial on Principal Component Analysis by Jonathon Shlens and chapter 8 in *Applied Multivariate Statistics* by Richard A. Johnson and Dean W. Wichern (available at the MDH library). Some ideas for applications and suggestions for sources on data for a specific field can be found using the Principal component analysis page on Wikipedia. There is also a short example in the compendium.

Some things you could choose to look at:

- Apply PCA to some real data and analyse the result. Is PCA suitable for this data? Do you see any potential problems?
- Make a comparison of PCA using Singular value decomposition (SVD) and the covariance/correlation matrix method. When is it ok to use the covariance or correlation matrix? Can you find an example where the two methods give large differences in the result?

1.2 Recursive Least Square and the Kalman Filter

The least square method (LSM) is a very popular and efficient way to find a function that fits a data set in an optimal way. Now consider what would happen if the data set would change over time and the optimized function needs to be optimized dynamically. Redoing the least square fit over and over again quickly becomes inefficient. There are ways to mimimize the amounted of required calculations though, one of the is using the *recursive least squares* method.

A further problem that can appear in real applications is that measurements are subject to noise and measuring the same quantity several times will not necessarily yield the same result. A common method to reducing this problem is using a *Kalman filter* which is a digital filtering technique that can be described very efficiently with matrices.

Both of these problem appear in guidance and navigation systems for vehicles and time series econometrics. The Kalman filter in particular is also related to control theory, see section 4.3.

A good general description of recursive least squares and the Kalman filter and how they relate to each other is given in *Excerpt from 'Introduction to Applied Mathematics'* by Gilbert Strang (can be found under 'Sources for projects' in Blackboard).

- In this project you can choose to either give an in-depth description of recursive least squares and/or a Kalman filter, or you can give a less in-depth description and implement either method using some suitable programming language.

1.3 Partial Least Squares Regression

Partial Least Squares Regression (PLS) is a method that is used to analyze and interpret stochastic or noisy data just like PCA or LSM and was originally developed for economy research but is today used in many other areas as well including chemometrics and medical imaging.

A good introduction to PLS can be found in Partial Least Square Regression by Hervé Abdi.

- In this project you can choose to either give a detailed description of PLS or give a shorter description of PLS and implement it using a suitable programming language.

1.4 Calculating the exponential matrix

The exponential matrix is a useful tool for solving matrix differential equations. Usually it is defined as follows:

$$e^A = \sum_{k=0}^{\infty} \frac{1}{k!} A^k$$

Since this is an infinite sum actually finding the elements of e^A is not trivial. How can this be done?

There are many different ways to answer this question, here are some suggestions

1.4.1 Analytical calculation of the exponential matrix:

Find a few cases where a simple (finite) analytical expression for the elements in e^A can be found. For example, what is e^A if A is a diagonal matrix? What about when a matrix can be decomposed into Show how these expressions can be found for some interesting cases. A recommmended place to start would be examining different kinds of factorizations for the matrices.

1.4.2 Numerical calculations of the exponential matrix:

There are many different ways of approximating the exponential matrix but finding a method that is good in general is difficult. Find one or a few different methods and discuss how they work, when they work and their advantages and disadvantages. A good place to start is Nineteen Dubious Ways to Compute the Exponential of a Matrix, Twenty-Five Years Later by Cleve Moler and Charles van Loan.

1.4.3 Calculating the exponential matrix using a quantum computer:

A very active research area is quantum computation, a new form of fundamental computer design based on the theory of quantum mechanics that open up possibilities for new methods and algorithms. One of the earliest discoveries was that a quantum computer would be able to simulate quantum mechanics much more efficiently than a classical computer could. This is largely due to the fact that there is a very efficient way of approximating a certain kind of exponential matrix that is involved in simulating quantum mechanics. Discuss how this method works and why it is better than classical algorithms. For more information see Simulating Physics with Computers by Richard Feynman, Efficient Simulation of Quantum Systems by Quantum Computers by Christof Zalka and Universal Quantum Simulators by Seth Lloyd.

1.5 Theoretical description of other method

- Choose some other method mentioned in another project and describe it thoroughly from a mathematical point of view.

1.6 Linear programming

Often in applications we might have a system of linear inequalities such as $x + y \leq 2$ rather than a system of linear equations. We are then usually interested in maximizing (or minimizing) a function on these variables with constraints given by the inequalities. We can write the problem as:

Maximize the function

$$p = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

where $c_1, c_2, \ldots, c_n$ are known constants and $x_1, x_2, \ldots, x_n$ are subject to the constraints

$$\begin{array}{ccccccccc} a_{1,1}x_1 & + & a_{1,2}x_2 & + & \dots & + & a_{1,n}x_n & \leq & b_1 \\ a_{2,1}x_1 & + & a_{2,2}x_2 & + & \dots & + & a_{2,n}x_n & \leq & b_2 \\ \vdots & & \vdots & & & & \vdots & & \vdots \\ a_{m,1}x_1 & + & a_{m,2}x_2 & + & \dots & + & a_{m,n}x_n & \leq & b_m \end{array}$$

These kind of problems are common in operations research as well as in microeconomics and company management when optimizing for example production and transportation of goods.

One common method to used to solve such a problem is the Simplex algorithm. Some material on the method can be found in "Excerpt from Elementary Linear Algebra, With Applications" by W.Keith Nicholson, there is also a good page in Wikipedia Linear programming.

1.6.1 Apply linear programming

- Find a real situation where linear programming might be useful. Explain why and how you use the method and interpretate the results. An example of an application is given in 5.2.

1.6.2 Describe linear programming

- Explain in more detail how and why linear programming / the simplex method works, what kinds of problems it can be used for and if you can what advantages/disadvantages it has over other methods.

1.7 Resistance distance, the Laplacian matrix and applications

The resistance distance between two vertices in a graph is commonly used to calculate the resistance between two points in an electrical network. There are however other applications such as a measure of "cyclicity" for molecules in chemistry.

The resistance distance between two vertices v_i, v_j in a graph G is defined as the effective resistance between them when each edge is replaced with a 1 Ω resistor. We define

$$\Gamma = \mathsf{L} + \frac{1}{n}$$

Where L is the laplacian matrix, n is the number of vertices in G and 1 is a $n \times n$ matrix with all elements equal to one.

The elements of the resistance distance matrix (Ω) is then:

$$(\Omega)_{i,j} = \Gamma^{-1}_{i,i} + \Gamma^{-1}_{j,j} - 2\Gamma^{-1}_{i,j}.$$

- Explain how to calculate resistance-distance and how it relates to the spectrum of the Laplacian matrix.

- Show how resistance-distance can be used in an application such as for electrical networks or in chemistry.

Some information on resistance-distance can be found on Wikipedia Resistance distance. More information, in particular how it relates to the eigenvalues and eigenvectors of the laplacian matrix can be read in for example Resistance distance and Laplacian spectrumFor additional information as well as an application in chemistry you can look at Resistance-distance matrix: A computational algorithm and its application

2 Mathematics

2.1 The Vandermonde Matrix determinant

A *Vandermonde* matrix is a matrix that looks like this

$$V = \begin{bmatrix} 1 & x_1 & x_1^2 & \dots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \dots & x_2^{n-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^{n-1} \end{bmatrix}$$

There is a relatively simple formula for the determinant of this matrix.

$$\det(V) = \prod_{1 \leq i \leq j \leq n} (x_j - x_i)$$

In this project we want you to show and explain a proof of this formula. There are two wellknown proofs, one by Donald E. Knuth and one by Leon Mirsky. Both of these proofs can be found on ProofWiki.

2.2 The inverse of the Vandermonde matrix

In the previous project a relatively simple formula for the determinant of the Vandermonde matrix was given. As it turns out you can find a formula of similar difficulty for the inverse of the Vandermonde matrix. This inverse is described in Inverses of Vandermonde matrices by N. Macon and A. Spitzbart. Variations on the Vandermonde matrix can be created by only including even or odd powers

$$U = \begin{bmatrix} 1 & x_1^2 & x_1^4 & \dots & x_1^{2n} \\ 1 & x_2 & x_2^2 & \dots & x_2^{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & x_n^2 & x_n^4 & \dots & x_n^{2n} \end{bmatrix}$$

$$W = \begin{bmatrix} 1 & x_1 & x_1^3 & \dots & x_1^{2n-1} \\ 1 & x_2 & x_2^3 & \dots & x_2^{2n-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & x_n & x_n^3 & \dots & x_n^{2n-1} \end{bmatrix}$$

If the determinant and inverse of the Vandermonde matrix is known it is also simple to find the determinant and inverse for these matrices, you can see how this can be done in *Excerpt from 'Some Eclectic Matrix Theory'* by Miller (can be found under 'Sources for projects' in Blackboard). As a project you can choose some interesting formulas from the mentioned sources and describe in detail how they can be derived.

The Vandermonde matrix are also important in some areas of mathematical finance see project 3.3.

2.3 Matrix equations

In mechanics, quantum physics, computer science, control theory, statistics and solid state physics there are many important applications and powerful methods based on properties of solutions to matrix equations. The most important examples of matrix equations are of the formal

$$\begin{aligned} AX &= XB, \\ AX - XB &= C, \\ AXB - CXD &= E, \end{aligned}$$

In this project you can use different techniques to solve this kind of equations.

Techniques include using the Kroenecker product to rewrite the matrix equations on the standard $Ax = y$ form, or techniques that work for more specific equations, as described in 'Solution of equation $AX + XB = C$ by inversion of an $M \times M$ or $N \times N$ matrix' by Anthony Jameson.

- Choose one, or more, of the equations above. Either you can make a detailed examination of the mathematics or you can demonstrate how to use the techniques practically by solving an interesting problem. Either a constructed or a problem taken from an application.

3 Finance & Economics

3.1 Input-Output analysis in economics

Input-Output models such as the Leontief model presented in the compendium are important when describing the economy on a large scale, such as for a country. An introduction to the model can be found in the compendium, you can also read more in 'Nonnegative matrices in the mathematical sciences' by Abraham Berman and Robert J. There is also a page in Wikipedia which might have further information or references: Input-output model.

- Describe the model and try it on some sample data, either fictionary or if you can find some real data to use it on. Other things to consider could be to look at what limitations the model have and what alternative or generalized methods are there?

3.2 The Bellman equation and dynamic programming

Dynamic programming (the mathematical method not the programming method) is a method in which we can solve certain optimization problems by breaking it up in multiple small parts, each of which we can solve individually. Generally we need the problem to have "optimal substructure", which means that the optimal solution can be constructed efficiently from optimal solutions of it's subproblems.

These type of problems often arise in Markov decision processes where we want to maximize a reward function over times. Where the process is described by a Markov chain, which in turn is affected by a policy which decides how to act in every state.

You can get an introduction to dynamic programming in wikipedia Dynamic programming and bellman equation. There is also a chapter on dynamic programming in "Introduction to Matrix Analysis" by Richard Bellman. You can find this in "Excerpt from Introduction to Matrix Analysis".

- Describe how the method works in theory and give some practical real or fictionary examples where you show how to use it.

3.3 Vandermonde's matrix in mathematical finance

The Vandermonde matrix is described in the lecture notes, and in project 2.1, and it can be very useful in several different areas including mathematical finance. In the article 'On the Vandermonde matrix and its role in mathematical finance' by Ragnar Norberg the relevance of the Vandermonde matrix in several financial applications, including zero coupon bond prices and situations where the spot rate of interest is modeled as a continous time, homogenous, recurrent Markov chain.

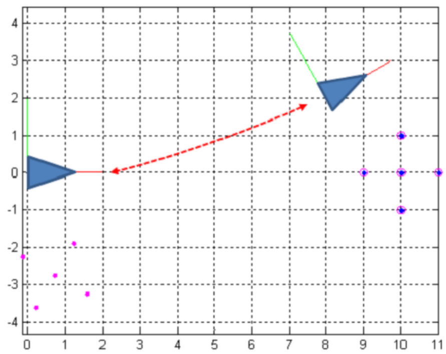
- Choose one of the applications you find interesting and describe how the Vandermonde matrix appears in the application and what properties of the Vandermonde matrix, or some related matrix, are important.

4 Robotics, Aviation & Control

4.1 Computer vision

4.1.1 Egomotion estimation

The problem of estimating the robot motion given a set of measurements on the 3D environment in two subsequent instants of time is know in robotics as "egomotion" estimation. Suppose to have a robot moving on a planar surface and a sensor (a camera or a laser) mounted onboard that can measure the position of interest objects in (X Y) coordinates. From two subsequent sets of measurements of at least two objects, it is possible to estimate the motion of the robot through a proper matrix formalization.

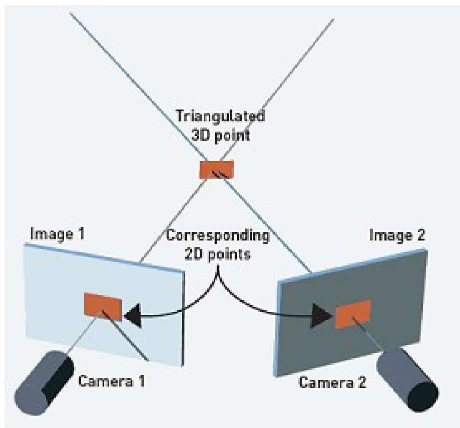


- Study 'Validation of Stereo Matching for Robot Navigation' by J. Lidholm, G. Spampinato and L. Asplund and present how this matrix formalization works.

4.1.2 Stereo Triangulation

The problem of estimating the position of an object in the 3D environment can be solved in robotics using a distance sensor. In case the robot is equipped only with cameras, the problem can be solved only if two cameras are used simultaneously. Taking two pictures of the same object at the same time by a stereo camera (a sensor containing two cameras) it is possible to retrieve the position of the object in 3D by a process that, in computer vision, is called *stereo triangulation*.

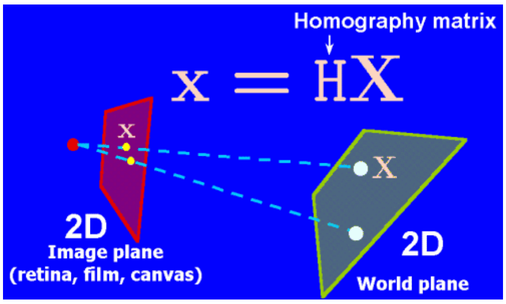
This process is described by Jean-Yves Bouguet 'Stereo Triangulation in Matlab' which is available on Blackboard.



- Describe how this method works in detail. Some further information can be found on the course page for '3D Photography' at www.multires.caltech.edu.

4.1.3 Planar Homography

The relation between an object in 3D and its corresponding image taken by a single camera is not biunique since three coordinates are needed to describe an object in the 3D space while only two are required to identify its corresponding projection in the image of the camera. In practice this means that given a 3D object it is always possible to find its corresponding projection in the image, but not vice versa. It is possible to reconstruct a 3D object given the corresponding image coordinates, if two images are available for a stereo camera or if the 3D object lies in a plane. In this last case the relation between and the object and its corresponding projection is called, in computer vision, planar homography and is represented by a 3×3 homography matrix. The method to retrieve this matrix given at least five corresponding points is called *Direct Linear Transformation*.



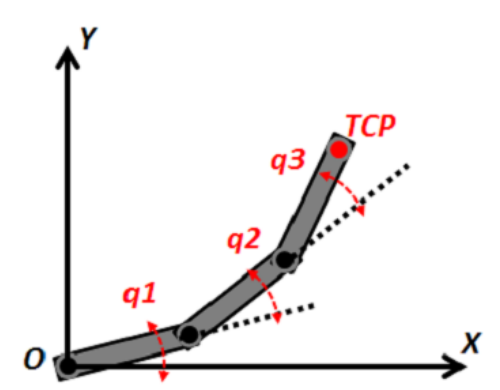
- This method is described in 'RANSAC for dummies' appendix C by Marco Zuliani. Describe how this method works.

4.2 Robotic control

More information on the following projects can be found in 'Robotics: modelling, planning and control' by B. Siciliano, L. Sciavicco, L. Villani and G. Oriolo.

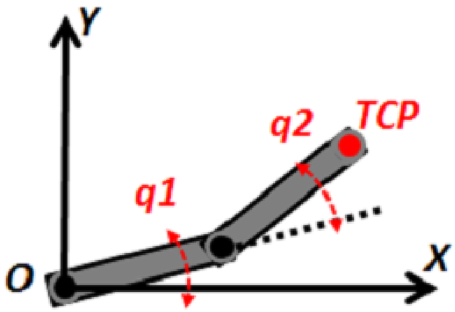
4.2.1 Forward kinematics

Suppose to have the three joints robot shown in the picture. Find the coordinates X and Y of the robot hand (TCP: Tool Centre Point) for given values of the joints $q1$, $q2$, $q3$. This problem is known in robotics as 'Forward kinematics'.



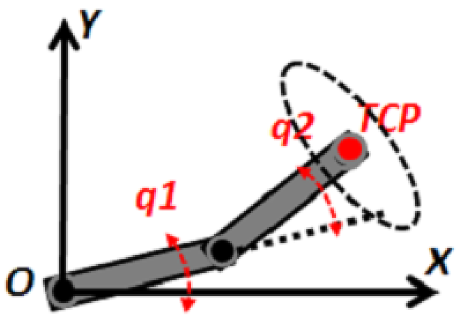
4.2.2 Singularity positions

Suppose to have the two joints robot shown in the picture. Suppose also to have the 2×2 Jacobian matrix J that puts into relations the joints velocities with the TCP velocities in the X and Y directions. Find the joints values that make zero the determinant of the matrix J. This problem is known in robotics as 'Singularity positions of the robot'.



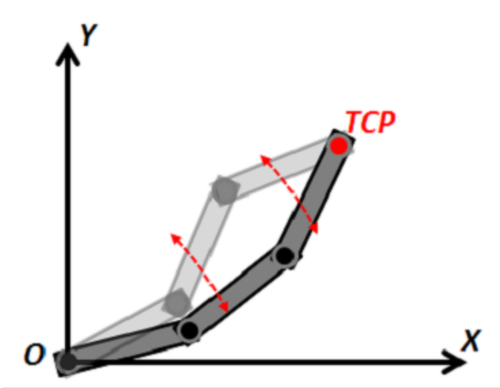
4.2.3 Manipulability analysis

Suppose to have the two joints robot shown in the picture. Suppose also to have the 2×2 Jacobian matrix J that puts into relations the joints velocities with the TCP velocities in the X and Y directions. Find the preferred directions of motion of the TCP using the SVD (Singular value decomposition) of the matrix J. This problem is known in robotics as 'Manipulability analysis of the robot'.



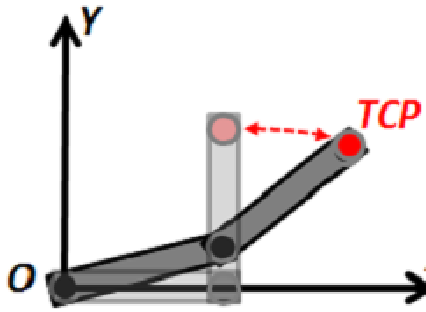
4.2.4 Redundancy analysis

Suppose to have the three joints robot shown in the picture. Suppose also to have the 2×3 Jacobian matrix J that puts into relations the joints velocities with the TCP velocities in the X and Y directions. Find the relation between the joint $q1$, $q2$ and $q3$ that leaves the position on the TCP unchanged, using the null space of the matrix J. This problem is known in robotics as 'Redundancy analysis of the robot'.



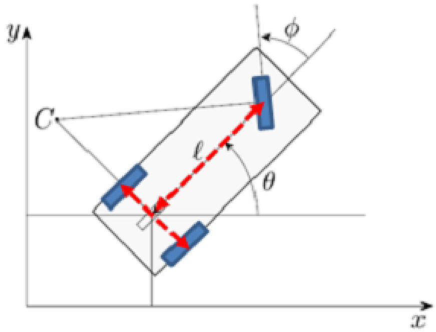
4.2.5 Iterative inverse kinematics

Suppose to have the two joints robot shown in the picture. Suppose also to have the 2×2 Jacobian matrix J that puts into relations the joints velocities with the TCP velocities in the X and Y directions. Find the joints positions corresponding to a given TCP position making an iterative gradient descendent least square optimization, using the matrix J as gradient. This problem is known in robotics as 'Iterative inverse kinematics of the robot'.



4.2.6 Non holonomic motion analysis

Suppose to have the tricycle robot shown in the picture. Suppose also to have the Pfaffian constraint matrix. Find the equations of the robot motion using the null space of the constraints matrix A. This problem in known in robotics as 'Non holonomic motion analysis of the mobile robot'.



4.3 Control Theory & Automatic Control

Control theory is the theory of how to describe how and if we can alter the behaviour of different systems in a desired way. Control theory has been very successfull in describing how linear systems can be controlled. A linear system is typically defined by the following equations:

$$\frac{\mathrm{d}x}{\mathrm{d}t}\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t) + D(t)u(t)$$

where x is the *state vector* with dimension n, y is the *output vector* with dimension q and u is the *control vector* with dimension q. These three vectors are related to each other via the A $(n \times n)$, B $(n \times p)$, C $(q \times n)$ and D $(q \times p)$ matrices.

A surprisingly large amount of different real systems can be described like this, many systems that are mainly thought of as non-linear can often be linearized (approximated by linear systems) or it can be possible to find a linear representation of them, for example by using a transform technique such as Laplace transform.

There are many different ways to analyze linear systems by analyzing the matrices that describes them. Below you can find some examples of this.

Controllability

Consider a linear system described as in section 4.3. Let the initial state of the system be represented by $x(0)$.

Suppose we want to put this system in a certain state $x(t_f)$, is it possible to choose a control signal u that will transfer the system from state $x(0)$ to state $x(t_f)$ in a finite amount of time, $0 < t \leq t_f$?

If it is possible, the system is said to be *controllable*. There are ways to check the *controllability* of a system by analyzing the two matrices A and B. On page 617-621 in 'Control Systems Engineering' by I.J. Nagrath and M. Gopal (excerpt available on Blackboard, book available at the library) two methods, called Gilbert's method and Kalman's test are described.

4.3.1 Controllability in an application

- Find a real example of a state matrix from an application and describe how the tests can be used to determine controllability.

4.3.2 The B-matrix and controllability

- Choose a matrix B and vary it in different ways and see how this affects the controllability. If you want to you can also examine what happens when you change the A-matrix.

Observability

Consider a linear system described as in section 4.3. Let the initial state of the system be represented by $x(0)$.

Suppose we can observe the output from this system for a period if time $0 \leq t \leq t_f$. If these observations, $y(t)$, can be used to find the systems initial state, $x(0)$, the system is said to be *observable*. There are ways to check the *observability* of a system by analyzing the two matrices A and C. On page 621-625 in 'Control Systems Engineering' by I.J. Nagrath and M. Gopal (excerpt available on Blackboard, book available at the library) a method for doing this is described.

4.3.3 Observability in an application

- Find a real example of a state matrix from an application and describe how to determine observability.

4.3.4 The C-matrix and observability

- Choose a matrix C and vary it in different ways and see how this affects the observability. If you want to you can also examine what happens when you change the A-matrix.

4.3.5 Dynamic systems and eigenvalues

A second order dynamic system is described by the differential equation

$$\dot{\mathsf{X}}(t) = \mathsf{A}\mathsf{X}(t)$$

in which the matrix A is square $n \times n$ and $\mathsf{X}(t)$ is a column vector of length n.

Consider the case where the matrix A is 2×2.

Study the *phase portrait* of the system, that is how the first element in X depends on the second element in X, as well as the evolution in time of the solution in the following cases of eigenvalues of A:

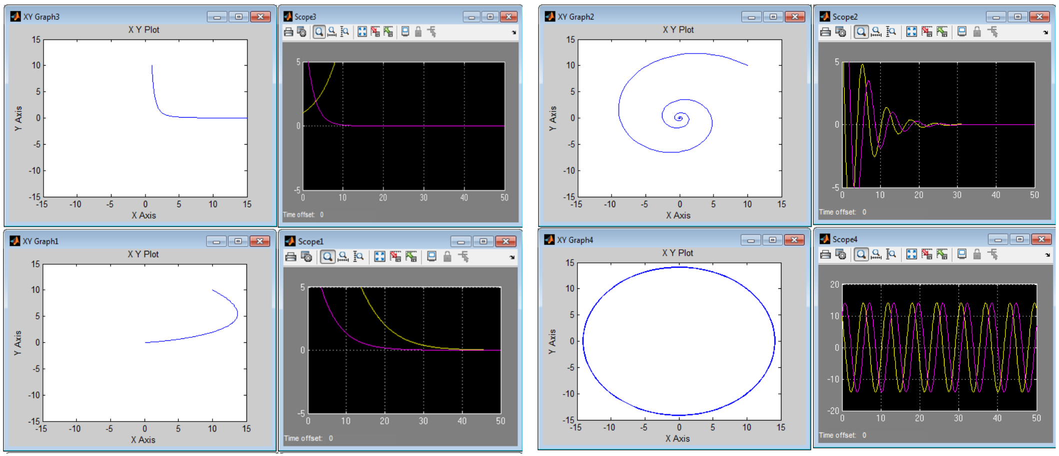
- real and distinct both negative

- real and distinct one negative and one positive
- real and coincident
- complex and conjugate with real part negative
- complex and conjugate with real part zero

More information about small dynamical systems can be found in 'Continuous dynamical systems' by Lars-Erik Persson.

Also consider the case where $n > 2$. All the previously studied cases can be seen as comparisons between pairs of elements from X. Is there anything general to be said about the behaviour of systems where n is odd?

It is recommended to use some suitable tools to numerically calculate the behaviour of the systems.



4.3.6 Liapunov Stability

While controllability and observability are desirable properties they are not always necessary. A system, or part of a system, does not have to be controllable or observable to be predictable and manageable. If a system is *stable* there is some state $\tilde{x}$, called the *equilibrium state* (or equilibrium point), such that for any initial state $x(t_0)$ that is close enough to $\tilde{x}$ it will stay within a certain distance of $\tilde{x}$ for all $t > t_0$.

A popular formal definition of stability was given by A.M. Liapunov and based on this definition he gave a number of theorems regarding the stability (and instability) of different systems (non-linear as well as linear). One of these theorems state that if we can construct a function, $V(x)$, with certain properties the system is stable. $V(x)$ is called the *Liapunov function* and a simple but widely useful method of constructing $V(x)$ is called *Liapunovs direct method*. For linear systems the crucial part of Liapunovs direct method involves solving the matrix equation:

$$A^\top P + PA = -Q$$

Where Q is an arbitrary symmetric positive definite matrix and P is an unknown symmetric positive definite matrix for which we want to solve the equation.

Details and examples of Liapunovs direct method can be found on page 642-660 in 'Control Systems Engineering' by I.J. Nagrath and M. Gopal (excerpt available on Blackboard, book available at the library).

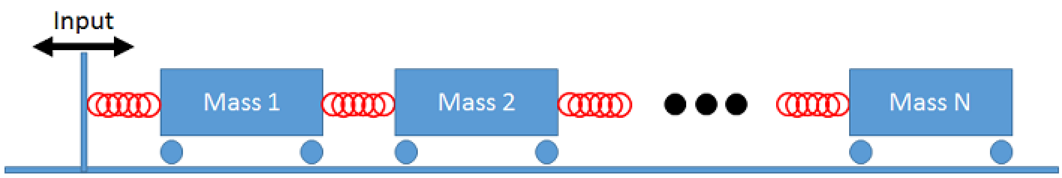
- Describe Liapunovs direct method and either give a thorough proof that it works or demonstrate how to use the method. The demonstration can either be on a general system (called *Krasovskii's method* in 'Control Systems Engingeering') or on an application of your choice.

4.3.7 Adaptive flight control for an F-18 Aircraft

- A real example of Liaponuv stability analysis can be found in 'Adaptive Flight Control Design with Optimal Control Modification on an F-18 Aircraft Model' by J. Burken, N. T. Nguyen and B. J. Griffin. In this report the plane is modelled using matrices and several different kinds of analysis are performed. For the most important matrices, describe which different properties are examined, or choose a single type of analysis used and describe it in detail.

4.3.8 Modelling and simulation of a train of masses

A train of N masses are constrained to move on a flat surface with a friction coefficient B. the masses are connected to each other by springs of elastic coefficient K. The first spring is anchored to a wall which can change position and represents the input to the whole system.

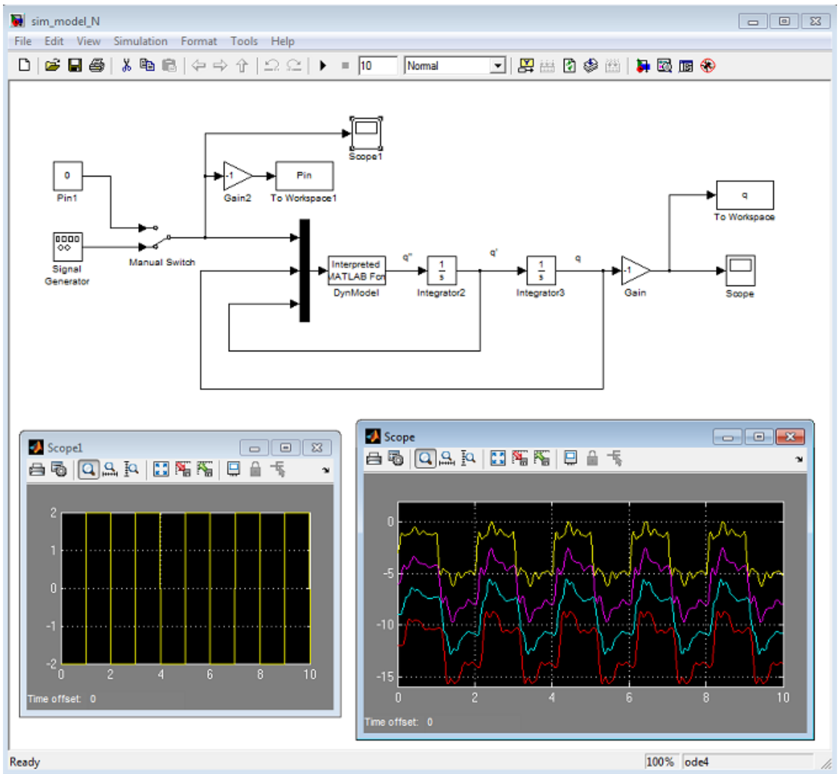


Find the matrices A and B and C which describes the dynamics of the system as a set of differential equations on the form:

$$\ddot{\mathsf{q}}(t) = \mathsf{A}\mathsf{q}(t) + \mathsf{B}\dot{\mathsf{q}}(t) + \mathsf{C} \cdot Input$$

in which q represents the vector containing the positions of each masses.

Solve the system of differential equation in simulation using Simulink.



sim_model_N
File Edit View Simulation Format Tools Help
Normal
Pin1
Signal Generator
Manual Switch
Gain2
Pin
To Workspace1
Scope1
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Gain
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Scope
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Ready
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ode4

5 Energy and electromagnetics

Apart from the suggestions here, the two suggestions "Principal component analysis" and "Partial least squares regression" under general topics have many applications in energy related problems as well.

5.1 Solving the Dirichlet problem

There is a deep connection between random walks in two dimensions, voltage in electrical network, temperature distributions and the two dimensional Dirichlet problem. Consider a problem given by Laplace equation:

$$\frac{\partial^2 f}{\partial x} + \frac{\partial^2 f}{\partial y} = 0$$

With a fixed value on the boundary (absorbing states). We then seek $f(x, y)$ on the interior points.

The solution have the property that the value of $f(x, y)$ is equal to the average of it's immediate neighbors. $f(x, y)$ could be the temperature with a fixed temperature at the boundary or the voltage with some part of the boundary fixed at a certain voltage and the other grounded (non-negative boundary).

This problem can be solved in many different ways. We can look at it as a game where we consider the random walk starting at an interior point, where we gain a reward equal to the boundary value. We then seek the "fair" price to play this game. We can solve this by simulating this game (Monte Carlo) or considering the Markov chain describing the game. Other methods include the "method of relaxation" where we iteratively average untill we have a good enough solution, or setting up the correct linear system to solve.

You can either look at a real problem, set up, discretisize it and solve it using at least one method described below, Or you could make a more indepth study of one of the methods or Alternatively you could make a comparison of the different methods for some easy example, what advantages or disadvantages can you find using the different methods?

A brief overview of the different methods from a electrical network perspective can be seen on page 12-29 in 'Random walks and electric networks' by Peter G. Doyle and J. Laurie Snell.

5.1.1 Solving the problem using the Monte Carlo perspective

- Look at a real problem, formulate it as a game and solve it from a Monte Carlo perspective.

5.1.2 Solving the problem using Markov chain theory

- Look at a real problem as a random walk on a graph and solve it using Markov chain theory.

5.1.3 Solving the problem using the method of relaxation

- Look at a real problem and solve the problem using the method of relaxation. Look at the Jacobi method and how to solve these kinds of problems with it. You can read more on the Jacobi method on Wikipedia.

5.1.4 Comparison of methods

- Briefly describe at least two different methods and make a comparison between them. Things to consider are for example how good the solution is and how fast the method gets there. But it can also be interesting to consider ease to set up and implement the method, or other interesting things you find.

5.1.5 Theoretical examination

- Look more indepth at one of the methods considered here, describe both how it works and some why it works. A small section describing harmonic functions and why they are interesting in this problem might be a good idea as well.

5.2 Sustainable energy using linear programming

In section 1.6 a method called *linear programming* is described. This method can be used to optimize systems with regards to a set of linear inequalities. This can be useful in many different applications, for example finding the most sustainable way of generating energy. One example of this can be found in 'A Linear Programming model for the optimal assessment of Sustainable Energy Action Plans' by Gianfranco Rizzo and Giancarlo Savino.

Show how they used linear programming to assess the energy plans.

5.3 Least-Squares Method and Application to Power Engineering, Electromagnetic Compatibility (EMC), Biomedical Engineering, etc.

Modelling of wire structures embedded in lossy media has received considerable attention in the last decades. Studies dealing with the electromagnetic coupling to telecommunication cables (EMC), design of grounding systems (Power Engineering), or studies of electrical stimulation of nerves (Biomedical engineering), are of interest. No matter what the application, in order to evaluate the current distribution along a wire conductor, which is buried in the semi-conducting medium (Fig. 1), a group of so-called Sommerfeld's integrals needs to be solved.

Let's consider the following form of the Sommerfeld's integral:

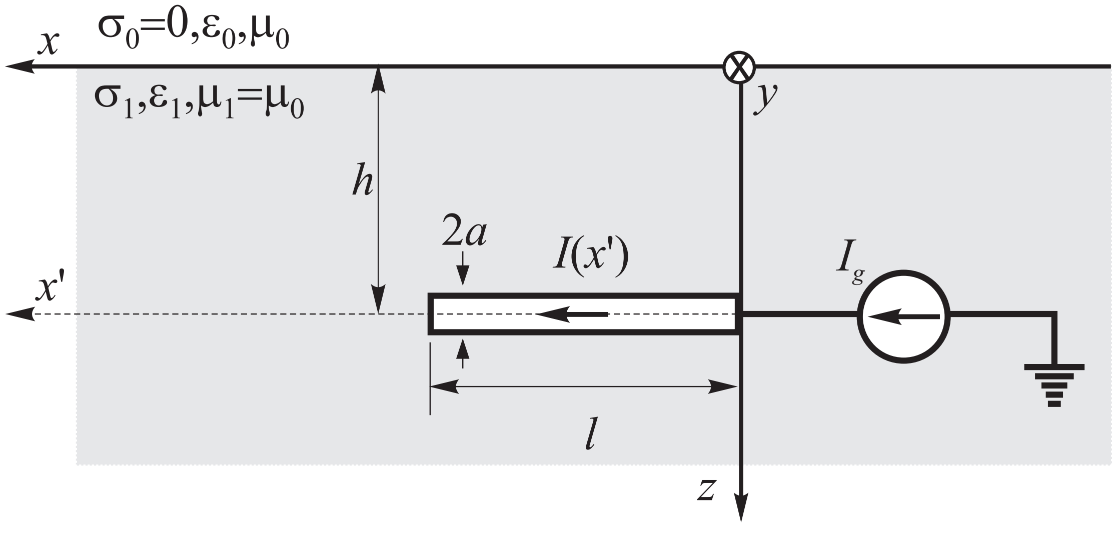


Figure 1: Schematic illustration of the wire ground electrode buried in the lossy half-space.

$$U_{11} = \int_0^{\infty} \tilde{T}_{\eta 1}(\alpha) \frac{\alpha}{u_1} J_0(\alpha\rho) \mathrm{d}\alpha, \tag{1}$$

where ρ - radial distance, $\underline{n}$ - refractive index (complex valued), J_0 - Bessel function of the 1st kind zero order, and

$$\tilde{T}_{\eta 1}(\alpha) = \frac{2u_1}{u_0 + u_1}, u_0 = \sqrt{\alpha^2 - 1}, u_1 = \sqrt{\alpha^2 - \underline{n}^2}. \tag{2}$$

Since this kind of integral doesn't have a closed form solution, a possible approach considers its approximation. We will adopt a solution proposed by Vujevic et al. in [1], which considers the approximation of the $\tilde{T}_{\eta 1}(\alpha)$ by:

$$\tilde{T}_{\eta 1}(\alpha) - 1 \approx \sum_{k=1}^{15} \underline{A}_k e^{-(u_1 - j\underline{n})D_k}. \tag{3}$$

where
$D_k \in \mathbf{R}$,
$D_k = D_{k-1} \sqrt[14]{10^6}, k = 2, ..., 15,$
$D_1 = 0.1 f_m$, and $f_m = \dfrac{0.795}{\sqrt{\mid 1 - \underline{n}^2 \mid}}$.

Using the LSM determine the complex constants $\underline{A}_k, k = 1, ..., 15$, so that (3) is satisfied in 25 sample points $\alpha_j, j = 1, ..., 25$, such that
$\alpha_j = \alpha_{j-1} \sqrt[23]{10^6}, j = 3, ..., 25,$
$\alpha_2 = \dfrac{10^{-5}}{f_m}$, and $\alpha_1 = 0$.

Using MATLAB or any other programming platform, calculate unknown complex constants $\underline{A}_k$ for the following values of the refractive index $\underline{n}$:

- $\underline{n} = 10 - j0.001$, corresponds to very lossy ground,
- $\underline{n} = 10 - j10$, corresponds to real ground,
- $\underline{n} = 10 - j600$, corresponds to perfectly conducting ground.

5.4 Least-Squares Method and Application to Modelling of Lightning Discharge

Mathematical modelling of lightning discharges is an important component in the process of risk estimation in lightning protection design, and prevention of failures and damage to power systems and electronic devices. Analysing different physical characteristics of lightning, efforts have been put into developing an appropriate procedure for modelling lightning electromagnetic field and discharge currents, either measured, or typical as described in the official standards for lightning protection. For engineering and electromagnetic models, a function for representing different lightning channel-base currents is needed, having not too many parameters, but still able to reproduce desired waveshapes.

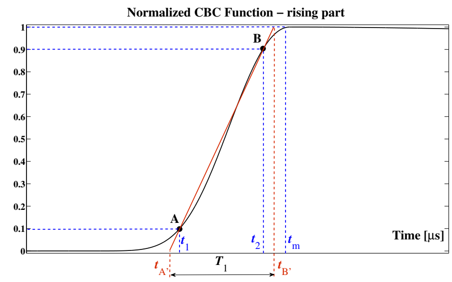


Figure 2: Lightning current representation of the rising part of the first positive stroke by the normalized CBC function.

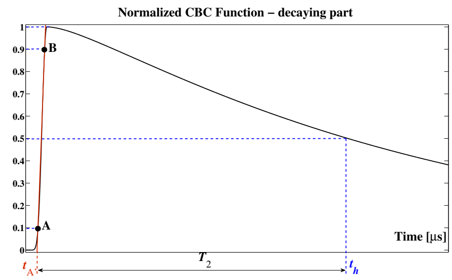


Figure 3: Lightning current representation of the decaying part of the first positive stroke by the normalized CBC function.

Using the LSM, evaluate the parameters of the lightning current function CBC (Channel Based Current) given by (4) and shown in (Fig. 2 and Fig. 3), so it fits the standard Heidler function given by (5).

Perform the analysis for the following waveshapes:

- First positive stoke (FPS): $I_0 = 200$ kA, $T_1/T_2 = 10/350$ μs,
- First negative stroke (FNS): $I_0 = 100$ kA, $T_1/T_2 = 1/200$ μs,
- Subsequent negative stroke (SNS): $I_0 = 50$ kA, $T_1/T_2 = 0.25/100$ μs,

where I_0 - max current value at moment t_m, T_1 - rising time, T_2 - decaying time.

$$i_{\mathrm{CBC}}(t) = \begin{cases} I_0(t/t_m)^a e^{a(1-t/t_m)} = I_0 f^a(t), \; 0 \le t \le t_m, \\ I_0(t/t_m)^b e^{b(1-t/t_m)} = I_0 f^b(t), \; t_m \le t \le \infty \end{cases} \tag{4}$$

$$i_{\mathrm{H}}(t) = \frac{I_0}{\eta}\frac{(t/\tau_1)^n}{1+(t/\tau_1)^n}e^{-t/\tau_2} \tag{5}$$

where η - peak current correction factor, τ_1, τ_2 and n - Heidler's function parameters.

So, our goal is to achieve

$$I_0 f^a(t_i) = i_{\mathrm{H}}(t_i), \tag{6}$$

at $t_i, i = 1, 2, ..., 10,$ equidistant sample points such that $0 \le t_i \le t_m$ and

$$I_0 f^b(t_j) = i_{\mathrm{H}}(t_j), \tag{7}$$

at $t_j, j = 1, 2, ..., 10,$ equidistant sample points such that $t_m \le t_j \le 2T_2$. Previous equations can also be written in a form

$$\ln(I_0) + a \ln(f(t_i)) = \ln(i_{\mathrm{H}}(t_i)), \tag{8}$$

$$\ln(I_0) + b \ln(f(t_j)) = \ln(i_{\mathrm{H}}(t_j)). \tag{9}$$

Evaluate the unknown parameters a and b of the CBC current function using the LSM, and adopting the parameter values of both functions given in Table 5.4 .

Heidler Function Parameters			
parameters	FPS	FNS	SNP
η	0.93	0.986	0.993
τ_1, [μs]	19	1.82	0.454
τ_2, [μs]	485	285	143
I_0, [kA]	200	100	50
n	10	10	10
CBC Function Parameters			
parameters	FPS	FNS	SNP
I_0, [kA]	200	100	50
t_m, [μs]	26	2.6	0.65

Table 1: Parameters of the Heidler and CBC functions for the observed waveshapes.

5.5 Antenna Theory

Numerous researchers have solved the problem of vertical and horizontal linear antennas in the air, or above ground, using various methods. The method of numerical solution of integral equations is most frequently used in strict analysis of mentioned antenna structures. One of the approaches is based on the so-called electric-field integral equation method, and formulation of the Hallén's integral equation. This equation is then solved for the current using an appropriate numerical method. The following projects consider application of the:

- Point-Matching Method,
- Least-Squares Method, and
- Galerkin-Bubnov Method.

5.5.1 Point-Matching Method Applied to Antenna Theory

Starting from the Hallén's integral equation (HIE) (10) for the symmetrical vertical dipole antenna (SVDA) in the free space (Fig. 4), calculate the current distribution along the antenna arm $I(z')$ applying the point-matching method. In this case, the Hallén's integral equation (HIE) is expressed as:

$$\int_0^l I(z')[K_0(r_1) + K_0(r_2)]\mathrm{d}z' - C_1 \cos(\beta_0 z) = \frac{U}{j60}\sin(\beta_0 z), \quad (10)$$

where l - length of an antenna arm, a - cross-section radius of an antenna arm, $\beta_0 = 2\pi/\lambda_0$, C_1 - unknown integration constant, $U = 1V$ - feeding voltage, $K_0(r_k) = \dfrac{e^{-j\beta_0 r_k}}{r_k}$, $k = 1,2$ - standard potential kernel, and $r_k = \sqrt{a^2 + (z + (-1)^k z')^2}$. Previous expression can be written in a normalized form, more appropriate for programming:

$$\int_0^1 I(u')[K_0(R_1) + K_0(R_2)]\mathrm{d}u' - C_1 \cos(Lu) = \frac{U}{j60}\sin(Lu), \quad (11)$$

where $u = z/l$, $u' = z'/l$, $L = \beta_0 l$, $K_0(R_k) = K_0(r_k)l = \dfrac{e^{-jLR_k}}{R_k}$, $k = 1,2$, $R_k = r_k/l = \sqrt{R_a^2 + (u + (-1)^k u')^2}$, $k = 1,2$, and $R_a = a/l$.

Assume the current in one of the following forms:

a) $I(z') = \sum_{m=0}^{M} I_m (\frac{z'}{l})^m$, or

b) $I(z') = I_0 + I_I e^{-j\beta_0(l-z')} + I_R e^{j\beta_0(l-z')}$,

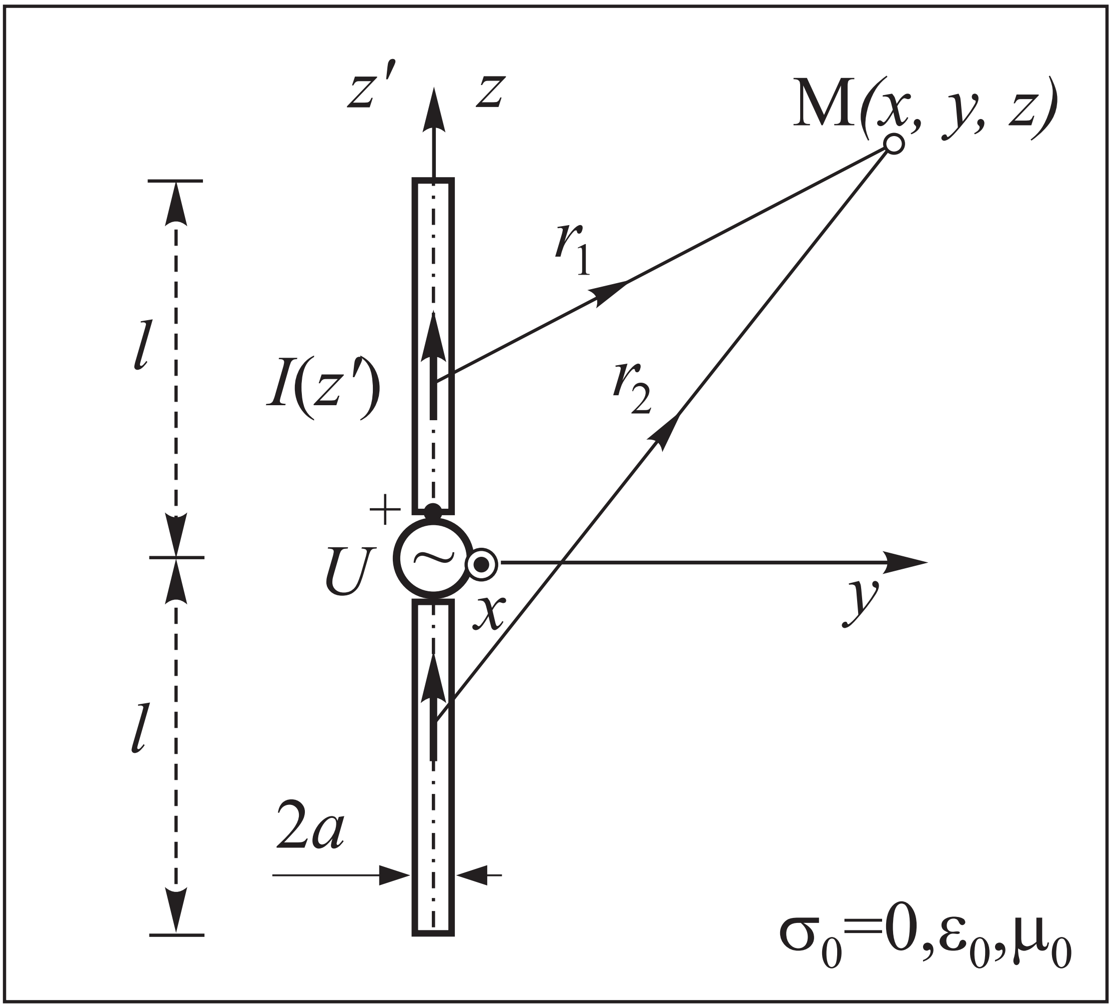


Figure 4: Schematic illustration of SVDA in the air.

or normalized:

a) $I(u') = \sum_{m=0}^{M} I_m (u')^m$,

b) $I(u') = I_0 + I_I e^{-jL(1-u')} + I_R e^{jL(1-u')}$.

Matching points should be chosen as follows:

a) $u_i = \dfrac{i-1}{M}, i = 1, ..., M+1$

This way, (M+1) linear equations are formed based on the HIE given by (11). One additional equation is needed since there are (M+2) unknowns - (M+1) current coefficients I_m and one integration constant C_1. The remaing equation is formed adopting the condition for the boundary value of the current at the end of an antenna arm. Usually, the vanishing of the current is assumed, i.e. $I(u'=1)=\sum_{m=0}^{M} I_m = 0$.

We now get the following system of linear equations:

$$\sum_{m=0}^{M} I_m \int_0^1 (u')^m [K_0(R_1(u_i)) + K_0(R_2(u_i))]\mathrm{d}u' - C_1\cos(Lu_i) =$$

$$= \frac{U}{j60}\sin(Lu_i), i = 1, ..., M+1, \tag{12}$$

$$\sum_{m=0}^{M} I_m = 0. \tag{13}$$

b) $u_i = \dfrac{i-1}{3}, i = 1, 2, 3.$

Here, one additional equation is needed to account for the unknown C_1 constant. Again, it should be assumed that the current is equal to zero at the end of an antenna arm, i.e. $i(u'=1) = I_0 + I_I + I_R = 0$. We now get the following system of linear equations:

$$\int_0^1 [I_0 + I_I e^{-jL(1-u')} + I_R e^{jL(1-u')}][K_0(R_1(u_i)) + K_0(R_2(u_i))]\mathrm{d}u' - C_1\cos(Lu_i) =$$

$$= \frac{U}{j60}\sin(Lu_i), i = 1, ..., 3, \tag{14}$$

$$I_0 + I_I + I_R = 0. \tag{15}$$

Now, solve one of the obtained systems of linear equations, and calculate the current distribution $I(u')$ along the antenna, and the input admittance of the antenna $Y_{in} = I(u'=0) = I_m, m = 0$ in [mS], for the following input data: $l/\lambda_0 = 0.25, 0.375, 0.5$, and $a/\lambda_0 = 0.007022$. For the case of the polynomial current distribution given by a), perform the calculations for the following polynomial orders: $M = 2, ..., 10$. Give graphic illustrations of the current distribution $I(u')$ for observed cases.

5.5.2 Least-Squares Method Applied to Antenna Theory

Starting from the Hallén's integral equation (HIE) (16) for the SVDA in the free space, calculate the current distribution along the antenna arm $I(z')$ applying the least-squares method. Assume the current in the following polynomial form $I(z') = \sum_{m=0}^{M} I_m (\frac{z'}{l})^m$, and take into account the boundary condition at the end of the antenna conductor, $I(z'=l)=0$.

The Hallén's integral equation (HIE) can be expressed as:

$$\int_0^l I(z')[K_0(r_1) + K_0(r_2) - 2K_0(r_0)\cos(\beta_0 z)]\mathrm{d}z' - \frac{U}{j60}\sin(\beta_0 z) = 0, \tag{16}$$

where l - length of an antenna arm, a - cross-section radius of an antenna arm, $\beta_0 = 2\pi/\lambda_0$, $U = 1V$ - feeding voltage, $K_0(r_k) = \dfrac{e^{-j\beta_0 r_k}}{r_i}$, $k = 0, 1, 2$ - standard potential kernel, $r_k = \sqrt{a^2 + (z + (-1)^k z')^2}, k = 1, 2$, and $r_0 = \sqrt{a^2 + (z')^2}$.

Previous equation can be written more compactly in the normalized form suitable for programming:

$$\int_0^1 I(u')[K_0(R_1) + K_0(R_2) - 2K_0(R_0)\cos(Lu)]\mathrm{d}u' - \frac{U}{j60}\sin(Lu) = 0, \tag{17}$$

or

$$\int_0^1 I(u')f(u, u')\mathrm{d}u' - \frac{U}{j60}\sin(Lu) = 0 \tag{18}$$

where $u = z/l$, $u' = z'/l$, $L = \beta_0 l$, $K_0(R_k) = K_0(r_k)l = \dfrac{e^{-jLR_k}}{R_k}, k = 0, 1, 2$, $R_k = r_k/l = \sqrt{R_a^2 + (u + (-1)^k u')^2}, k = 1, 2$, $R_0 = r_0/l = \sqrt{R_a^2 + (u')^2}$, and $R_a = a/l$.

The functional is formed in the following manner:

$$F = \sum_{i=1}^{N}[\int_0^1 I(u')f(u_i, u')\mathrm{d}u' - \frac{U}{j60}\sin(Lu_i)]^2 + CI(u' = 1) = 0, u_i = \frac{i}{N}, i = 1, ..., N, \tag{19}$$

or, after substituting the assumed current distribution

$$F = \sum_{i=1}^{N}[\int_0^1 \sum_{m=0}^{M} I_m (u')^m f(u_i, u')\mathrm{d}u' - \frac{U}{j60}\sin(Lu_i)]^2 + C\sum_{m=0}^{M} I_m = 0, u_i = \frac{i}{N}, i = 1, ..., N. \tag{20}$$

Unknown current coefficients $I_m, m = 0, ..., M$ and C are obtained from the following system of linear equations:

$$\frac{\partial F}{\partial I_m} = 0, m = 0, ..., M, \tag{21}$$

$$\frac{\partial F}{\partial C} = 0. \tag{22}$$

Now, solve the obtained system of linear equations, and calculate the current distribution $I(u')$ along the antenna, and the input admittance of the antenna $Y_{in} = I(u' = 0) = I_m, m = 0$ in [mS], for the following input data: $l/\lambda_0 = 0.25, 0.375, 0.5$, $a/\lambda_0 = 0.007022$, $M = 2, ..., 10$, and $N = 3, ..., 11$. Give graphic illustrations of the current distribution $I(u')$ for observed cases.

5.5.3 Galerkin-Bubnov Method Applied to Antenna Theory

Starting from the normalized form of the Hallén's integral equation (HIE) (17) for the SVDA in the free space, calculate the current distribution along the antenna arm $I(z')$ applying the Galerkin-Bubnov method. Assume the current in the following polynomial form $I(z') = \sum_{m=0}^{M} I_m (1 - \frac{z'}{l})^m$.

Starting point is the normalized HIE:

$$\int_0^1 I(u')[K_0(R_1) + K_0(R_2) - 2K_0(R_0)\cos(Lu)]\mathrm{d}u' - \frac{U}{j60}\sin(Lu) = 0, \tag{23}$$

and the assumed normalized current distribution $I(u') = \sum_{m=0}^{M} I_m (1-u')^m = \sum_{m=0}^{M} I_m f(u')$. Now, multiplying (23) by $f(u) = (1-u)^k$, and integrating over u from 0 to 1, we get:

$$\begin{aligned}\int_{u=0}^1 \int_{u'=0}^1 I(u')f(u)[K_0(R_1) + K_0(R_2) - 2K_0(R_0)\cos(Lu)]\mathrm{d}u'\mathrm{d}u = \\ = \frac{U}{j60}\int_{u=0}^1 f(u)\sin(Lu)\mathrm{d}u,\end{aligned} \tag{24}$$

or after substituting the current

$$\begin{aligned}\sum_{m=0}^{M} I_m \int_{u=0}^1 \int_{u'=0}^1 f(u')f(u)[K_0(R_1) + K_0(R_2) - 2K_0(R_0)\cos(Lu)]\mathrm{d}u'\mathrm{d}u = \\ = \frac{U}{j60}\int_{u=0}^1 f(u)\sin(Lu)\mathrm{d}u,\end{aligned} \tag{25}$$

where $f(u) = (1-u)^k$, $f(u') = (1-u')^m$, $m, k = 1, ..., M$.

Now, solve the obtained system of linear equations, and calculate the current distribution $I(u')$ along the antenna, and the input admittance of the antenna $Y_{in} = I(u' = 0) = I_m, m = 1$ in [mS], for the following input data: $l/\lambda_0 = 0.25, 0.375, 0.5$, $a/\lambda_0 = 0.007022$, and $M = 2, ..., 10$. Give graphic illustrations of the current distribution $I(u')$ for observed cases.

6 Computer Science

6.1 Pagerank and the power method

This project assumes assumes some knowledge in programming to complete successfully. Given A as the $n \times n$ adjacency matrix where non-zero rows are normalized to sum to one, a scalar $0 < c < 1$ usually $c = 0.85$, and column vectors e, g, v of length n. e is the vector with all elements equal to 1, g is zero for nodes with outgoing links and a 1 for all dangling nodes(node that doesn't link to any other node) and v is a non-negative weightvector usually e/n.

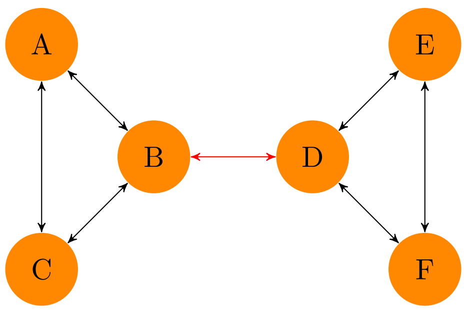
We define pagerank as the eigenvector with sum one to the eigenvalue equal to one of the matrix:

$$\mathsf{M} = c(\mathsf{A} + \mathbf{g}\mathbf{v}^\top)^\top + (1-c)\mathbf{v}\mathbf{e}^\top$$

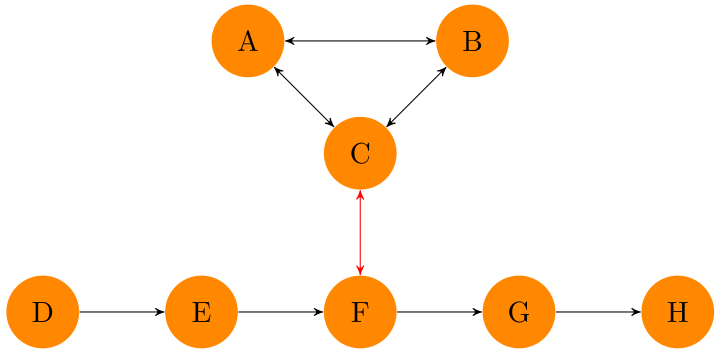
Since this is a positive vector (if $\mathbf{v} = \mathbf{e}/n$) and non negative we can use Perron-Frobenius to guarantee that this eigenvector exists.

Calculating this eigenvector is usually done using the power method where you iterate $\mathbf{p}_{n+1} = \mathsf{M}\mathbf{p}_n$ until it converges. Your project should include some of the theory behind PageRank as well as a short experimental study related to some of our research in the field where you implement and calculate PageRank for some example graphs.

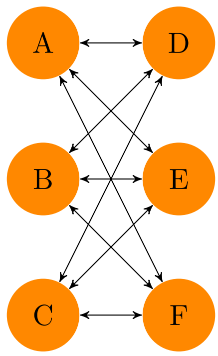
- Describe Perron-Frobenius theorem for non-negative irreducible matrices and its relevance to PageRank and the Power method.
- Look at one of the graph structures below and what happens when making small changes to the graph or parameters or take a closer look at the computational aspect in (d).
- a) Adding connections between two complete graphs (cliques). How does PageRank change when you add a single edge (in one or both directions) between them (adding the red edge between B and D)? Look at how the rank changes if you change the constant c. If you instead calculate all eigenvalues and eigenvectors, can you see anything else interesting as you do the above changes?



- b) Adding connections between one complete graphs (clique) and a line of vertices. How does PageRank change when you add a single edge (in one or both directions) between them (adding the red edge between C and F)? Look at how the rank changes if you change the constant c. If you instead calculate all eigenvalues and eigenvectors, can you see anything else interesting as you do the above changes?



- c) Looking at a bipartite graph. How does PageRank change when you change the number of vertices on one side or remove a single edge? Look at how the rank changes if you change the constant c. If you instead calculate all eigenvalues and eigenvectors, can you see anything else interesting as you do the above changes?



- d) A look at the computational aspect. Implement the algorithm with the necessary changes to accomodate for large matrices. Try it for some large systems, when does the "naive" method fail? Is it only the iterations taking longer or do you have to iterate more times as well to get the same accuracy? Note that you need to use sparse matrix representation (sparse(M) in Matlab) in order to gain in speed in the improved method.

You can read more on the mathematics behind pagerank at The \$25.000.000.000 Eigenvector, The linear algebra behind Google by K. Bryan and T. Leise, there is also a good page on pagerank and the power method on wikipedia. Another good article if you want to read more is Inside PageRank by M. Bianchini, M. Gori, and F. Scarselli.

6.2 The Householder transformation

Householder transformations are used extensively in numerical linear algebra to calculate QR-decomposition, transform to Hessenberg form or to tridiagonal form in the case of symmetric matrices.

A Householder transformation describes a reflection about a plane (or hyperplane in more dimensions) containing the origin.

The Householder matrix is described by:

$$P = I - 2vv^H$$

Where v is a unit vector ($||v|| = 1$) and H denotes the Hermitian transpose which is done by transposing the matrix and then taking the complex conjugate.

You can read more about Householder transformations how you can find them and what they are used for in 'Excerpt from Numerical Methods with MATLAB' by John Mathews and Kurtis Fink (can be found under 'Sources for projects' in Blackboard). You can also read more in Wikipedia householder transformation, QR-decomposition.

- How can you describe the Householder transformation geometrically? Look at one or a few areas where Householder matrices are used and give a short explanation how and why they are used there. For example you could look at the QR-method for computing the eigenvalues of a matrix and the importance of Householder transformations to speed up the calculations.