

Till skrivvakten: det ska ges information på engelska i skrivningslokalen. Det ska vara rutat papper som ska delas ut i skrivsalen.

This exam consists of six problems, each of which is worth 5 points. Marks:

Points	ECTS mark	Swedish mark
27–30	A	VG
22–26	B	VG
18–21	C	G
15–17	D	G
12–14	E	G

1. Find all the points on the parabola $y = x^2 - x + 1$ such that the tangent lines at those points pass through the origin. Write also equations of the lines in question.
2. Determine the dimensions of a container with shape of a cylinder, having the total area of 600π m² and largest possible volume. State even this maximal volume.
3. Compute the area bounded by the curves $y = \cos^2 x$ and $y = \sin^2 x$, if $0 \leq x \leq \pi/2$.
4. Show the convergence of $\int_1^{\infty} 2x^{-1}(x^2 + 1)^{-1} dx$ by computation.
5. a) Formulate the integral criterion for convergence of positive series (2 p.)
b) Determine the convergence of $\sum_{n=2}^{\infty} (n \ln n)^{-1}$ (3 p.)
6. Consider the function $f(x) = x(e^x - 1)^{-1}$. Compute the first three non-zero terms of it's Mac-Laurin expansion.

Good luck!